Nonreciprocal Two-Ports Represented by Modified Wheeler Networks*

H. M. ALTSCHULER† AND W. K. KAHN†

Summary—The extension of the (reciprocal) modified Wheeler network to include the more general nonreciprocal two-port is given. This representation is derived via a known decomposition of the general nonreciprocal network into two portions, one reciprocal, the other nonreciprocal. The reciprocal portion is then taken as the modified Wheeler network. Recombination of the elements results in the desired representation which is constituted of a minimum number, *i.e.*, of eight, passive elements. Each of these is a natural idealization of a physical microwave component. Since six of the elements belong to the class of "bilaterally matched" networks, some of the properties of this class are discussed. Two of the bilaterally matched elements embody the nonreciprocal properties of the network: a one-way attenuator and a one-way phase-shifter. Many of the characteristics of the (reciprocal) modified Wheeler network carry over directly to this nonreciprocal representation. The microwave measurement of the network parameters is also indicated.

Introduction

A DIRECT extension of a representation of reciprocal two-ports to the case of nonreciprocal two-ports has recently been described by Haus.[1] In his paper he has given the decomposition of the general nonreciprocal two-port into a reciprocal two-port and a nonreciprocal one with certain useful commutation properties. His choice of the Weissfloch network for the reciprocal portion has resulted in essentially two different representations. One of these, consisting of eight elements, is "mixed" in that the elements of the reciprocal portion are expressed in standard impedance (*i.e.*, resistance and reactance) terms, while the nonreciprocal portion is expressed in terms of its *ABCD* coefficients. The other representation indicated employs three gyrators in conjunction with ten impedance parameters and is by comparison with the first representation quite complicated. Clearly these parameters are not all independent of one another. Their interdependence is expressed by relatively complex equations.

What appears to be a more attractive representation, and one more meaningful to the microwave engineer, may be obtained in a manner similar to that employed by Haus, when the modified Wheeler network,[2] instead of the Weissfloch network, is used to represent the reciprocal portion. In that case the nonreciprocal elements which are employed are, with the exception of their "one-way" properties, conceptually similar to the reflection coefficient transformer (or ideal attenuator) and the transmission lines which form part of the reciprocal modified Wheeler network. In this sense the present network representation is "homogeneous." Furthermore, at any single frequency this network is highly suggestive of a procedure for synthesizing a nonreciprocal microwave two-port structure with the properties exhibited by the network. The network consists of the minimum number of elements required to represent a nonreciprocal two-port, *i.e.*, of eight independent elements which are the natural idealizations of physical microwave components.

All but two of the network elements are "bilaterally matched." Hence special attention has been paid to the class of bilaterally matched networks. In particular, a simple multiplication rule for their scattering matrices has been stated which permits the ready manipulation of such elements. Appendix II summarizes the various bilaterally matched elements which are of interest here, together with their scattering matrices and the network symbols used to represent them.

In the evaluation of particular two-ports as components for use in a microwave system, it is pointed to distinguish certain "essential properties" of the component from those which may be adjusted by the addition of lossless reciprocal networks (tuners) at the ports. These "essential properties" of a two-port, dissipation and nonreciprocal behavior, are explicitly exhibited by distinct bilaterally matched elements in the modified Wheeler representation. The ultimate limitations on the performance of a tandem connection of such components may then be inferred by inspection.

As in the case treated by Haus, the measurement of the network parameters here consists of the known measurement of the reciprocal (modified Wheeler) network in conjunction with the measurements necessary to obtain the additional nonreciprocal parameters.

The Ideal Amplifier Phase Shifter

The general linear, nonreciprocal, (not necessarily passive) two-port can always be decomposed[3] into a reciprocal two-port in tandem with a nonreciprocal two-port, as shown in Fig. 1 and (1). (See Appendix I for definitions of *A*, *B*, *C*, *D*).

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† Microwave Res. Inst., Polytechnic Inst. of Brooklyn, Brooklyn, N. Y.

[1] H. A. Haus, "Equivalent circuit for a passive nonreciprocal network," *J. Appl. Phys.*, vol. 25, pp. 1500–1502; December, 1954.

[2] H. M. Altschuler, "A method of measuring dissipative four-poles based on a modified Wheeler network," IRE Trans., vol. MTT-3, pp. 30–36; January, 1955; and "Representation and measurement of a dissipative four-pole by means of a modified Wheeler network," IRE Trans., vol. PGI 4, pp. 84–90; October, 1955.

[3] H. Schultz, "The transformation of the quadripole chain-matrix into diagonal form," *Arch. El. Übertr.*, vol. 5, pp. 257–266; June, 1951.

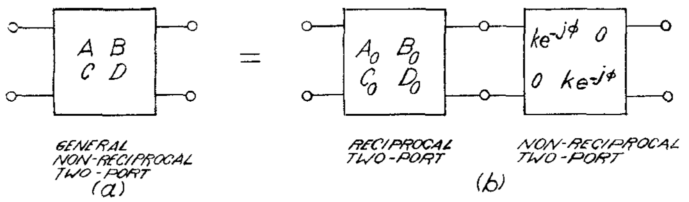


Fig. 1—Decomposition of general two-port.

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A_0 & B_0 \\ C_0 & D_0 \end{pmatrix} \begin{pmatrix} ke^{-j\Phi} & 0 \\ 0 & ke^{-j\Phi} \end{pmatrix}. \tag{1}$$

While this particular break-up is not the only one possible, it is unique when, in addition, it is required that the matrix representing the nonreciprocal two-port commute with *all* other matrices, *i.e.*, that it be a scalar matrix. It can be seen that this requirement can, in general, be met when one recalls that reciprocal two-ports must obey the condition

$$A_0D_0 - B_0C_0 = 1, \tag{2}$$

and when the matrix representing the general non-reciprocal two-port is decomposed as follows:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A_0 & B_0 \\ C_0 & D_0 \end{pmatrix} \begin{pmatrix} ke^{-j\Phi} & 0 \\ 0 & ke^{-j\Phi} \end{pmatrix},$$

where

$$\begin{pmatrix} A_0 & B_0 \\ C_0 & D_0 \end{pmatrix} = \left[\frac{1}{\sqrt{AD - BC}} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \right]$$

$$\begin{pmatrix} ke^{-j\Phi} & 0 \\ 0 & ke^{-j\Phi} \end{pmatrix} = \left[\sqrt{AD - BC} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right]. \tag{3}$$

This decomposition assures that $A_0D_0 - B_0C_0 \equiv 1$ and yields the definition $ke^{-j\Phi} \equiv \sqrt{AD - BC}$. The ambiguity in phase angle introduced by the square root is trivial.

The nonreciprocal two-port defined by the scalar matrix in (3) has been termed[1] "ideal amplifier phase shifter," and in a somewhat different context,[4] "ratio repeater." The scalar matrix $ke^{-j\Phi} \cdot I$ (where I is the unit matrix), of course, commutes with all other matrices. It follows that the ideal amplifier phase shifter represented by it "commutes" with *all* other networks; *i.e.*, its effect on the properties of a cascade of networks is completely independent of its position within the cascade.

For certain ideal or degenerate two-ports the terminal quantities (*e.g.*, V, I or a, b) at the in- and out-ports are not uniquely related. Examples are the one-way line ($S_{21} = 1$, $S_{12} = 0$) and the degenerate two-port consisting of two entirely separate one-ports. Transfer descriptions of such two-ports are singular. In terms of the scattering matrix and the particular transfer description employed (Appendix I) one has generally

[4] Harold A. Wheeler, "Generalized transformer concepts for feedback amplifiers and filter networks," Wheeler Monograph no. 5; August, 1948.

$$AD - BC = 0 \qquad \text{if } S_{12} = 0.$$

$$A, B, C, D \text{ do not exist if } S_{21} = 0.$$

The physical content of (3), and the discussions based on it which lead to the extended form of the modified Wheeler network, is preserved by a limiting argument. It is assumed that S_{21} and S_{12} are always finite, no matter how small.

By the application of formulas given in Appendix I, the scattering matrix for the ideal amplifier phase shifter can be shown to be

$$\begin{pmatrix} 0 & ke^{-j\Phi} \\ 1/ke^{-j\Phi} & 0 \end{pmatrix}. \tag{4}$$

This class of two-ports, *i.e.*, those described by scattering matrices with $S_{11} = 0$, $S_{22} = 0$, and their properties are of special interest and consequently discussed below.

Bilaterally Matched Networks

Two-ports characterized by zero values for the coefficients S_{11} and S_{22} are said to be "bilaterally matched." This implies that no reflections arise at the in-port when the out-port is terminated in its characteristic impedance and vice versa. The general (normalized) scattering matrix of this class of networks is given by

$$\begin{pmatrix} 0 & S_{12} \\ S_{21} & 0 \end{pmatrix}, \tag{5}$$

where S_{12} and S_{21} are arbitrary complex numbers. Of course, the ideal amplifier phase shifter is a member of this class [see (4)]. From the well-known input-output (voltage reflection coefficient) relation

$$\Gamma_{in} = \frac{(S_{11}S_{22} - S_{12}S_{21})\Gamma_{out} - S_{11}}{S_{22}\Gamma_{out} - 1}, \tag{6}$$

one sees, by inspection, that for bilaterally matched two-ports

$$\Gamma_{in} = S_{12}S_{21}\Gamma_{out}. \tag{7}$$

In view of the absence of any reflections at their junction, when two such two-ports, say

$$\begin{pmatrix} 0 & S_{12}' \\ S_{21}' & 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 & S_{12}'' \\ S_{21}'' & 0 \end{pmatrix},$$

are placed in tandem it is recognized that the scattering matrix of the tandem combination is

$$\begin{pmatrix} 0 & S_{12}'S_{12}'' \\ S_{21}'S_{21}'' & 0 \end{pmatrix}.$$

One may then define the following multiplication rule:

$$\begin{pmatrix} 0 & S_{12}' \\ S_{21}' & 0 \end{pmatrix} * \begin{pmatrix} 0 & S_{12}'' \\ S_{21}'' & 0 \end{pmatrix} = \begin{pmatrix} 0 & S_{12}'S_{12}'' \\ S_{21}'S_{21}'' & 0 \end{pmatrix}, \tag{8}$$

where this product, which will be referred to here as "star-product," applies specifically to the scattering matrices of bilaterally matched two-ports which have been placed in tandem. Eq. (8) also implies that any bilaterally matched two-port may be arbitrarily decomposed, in accordance with the star-product rule, into two or more bilaterally matched two-ports in tandem. In addition it follows from (8) that all two-ports or network elements in this class commute with each other, *i.e.*, the order of any two or more adjacent bilaterally matched two-ports may be interchanged without affecting the properties of the over-all network.

The (Reciprocal) Modified Wheeler Network

The modified Wheeler network, which is shown in Fig. 2, and methods for its measurement have already

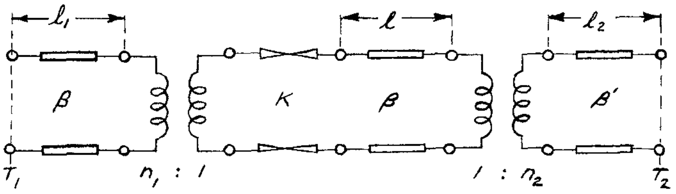


Fig. 2—Modified Wheeler network.

been described in detail elsewhere.[2] This network is a suitable representation of dissipative, *reciprocal*, passive, linear two-ports and consists of three lossless transmission lines l_1, l_2, and l, two ideal transformers n_1:1 and 1:n_2, and a "reflection coefficient transformer" K. The latter element[5] is an ideal attenuator in that its scattering matrix is

$$\begin{pmatrix} 0 & K \\ K & 0 \end{pmatrix}, \qquad 0 \leqq K \leqq 1. \tag{9}$$

The phase constant β of the transmission lines l_1 and l is that associated with the in-port, while β', the phase constant of line l_2, is associated with the out-port. It is also pertinent to point out that the scattering matrix of a lossless transmission line of electrical length θ is

$$\begin{pmatrix} 0 & e^{-j\theta} \\ e^{-j\theta} & 0 \end{pmatrix}. \tag{10}$$

Extension of the Modified Wheeler Network to the Nonreciprocal Case

The method used by Haus[1] in conjunction with the Weissfloch network to represent nonreciprocal two-ports can also be employed advantageously with respect to the modified Wheeler representation. Assuming the network break-up given in Fig. 1 and (1), one may represent the reciprocal two-port by the modified Wheeler network shown in Fig. 2. The ideal amplifier phase shifter $(ke^{-j\Phi}I)$ is then commuted to a new position between the elements K and l, and the three bilaterally matched elements $(K, ke^{-j\Phi}I, l)$ now located between the two transformers are examined as indicated below in conjunction with Fig. 3.

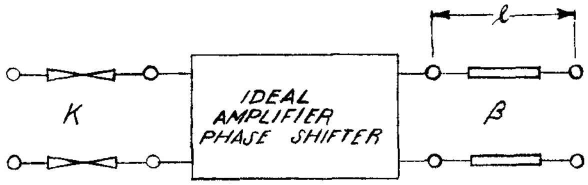


Fig. 3—Three bilaterally matched elements.

The three elements when taken together are represented by the star-product of their scattering matrices, [see (4), (9), and (10)] which when decomposed into a phase and a magnitude portion yields

$$\begin{pmatrix} 0 & Kk \\ K/k & 0 \end{pmatrix} * \begin{pmatrix} 0 & e^{-j(\theta+\Phi)} \\ e^{-j(\theta-\Phi)} & 0 \end{pmatrix}. \tag{11}$$

Since all the dissipation or amplification produced by the over-all network [such as in Fig. 1(a)] is expressed by the magnitude portion of (12), it is required that for *passive* networks this portion alone is also passive, *i.e.*, that the scattering parameters of the magnitude portion are subject to the condition

$$|S_{12}| = Kk \leqq 1 \text{ and } |S_{21}| = K/k \leqq 1. \tag{12}$$

Here K is restricted as indicated in (9) and k may consequently be larger than unity. In any case, one will have either $k \leqq 1$ or $1/k \leqq 1$. One can, therefore, demand a decomposition of the magnitude portion into two parts: one, a reciprocal passive element $(S_{12} = S_{21} \leqq 1)$; the other, a nonreciprocal passive attenuator element of such a type that complete transmission takes place in one direction (*i.e.*, either $|S_{12}| = 1$, or $|S_{21}| = 1$). Depending on the magnitude of k one then has

$$\begin{pmatrix} 0 & k^2 \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} 0 & K/k \\ K/k & 0 \end{pmatrix} \text{ for } k \leqq 1. \tag{13a}$$

or

$$\begin{pmatrix} 0 & 1 \\ 1/k^2 & 0 \end{pmatrix} * \begin{pmatrix} 0 & Kk \\ Kk & 0 \end{pmatrix} \text{ for } k \geqq 1. \tag{13b}$$

Analogously, the phase portion of (11) may be decomposed into

$$\begin{pmatrix} 0 & e^{-j2\Phi} \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} 0 & e^{-j(\theta-\Phi)} \\ e^{-j(\theta-\Phi)} & 0 \end{pmatrix} \text{ or} \tag{14a}$$

$$\begin{pmatrix} 0 & 1 \\ e^{j2\Phi} & 0 \end{pmatrix} * \begin{pmatrix} 0 & e^{-j(\theta+\Phi)} \\ e^{-j(\theta+\Phi)} & 0 \end{pmatrix}. \tag{14b}$$

There is little reason to choose one alternative over the other in (14).

It is now seen from (13) and (14) that the elements in Fig. 3 can be represented by the star-product

$$\begin{pmatrix} 0 & k^2 \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} 0 & K/k \\ K/k & 0 \end{pmatrix} * \begin{pmatrix} 0 & e^{-j2\Phi} \\ 1 & 0 \end{pmatrix}$$

$$* \begin{pmatrix} 0 & e^{-j(\theta-\Phi)} \\ e^{-j(\theta-\Phi)} & 0 \end{pmatrix}, \quad \text{for } k \leqq 1 \tag{15a}$$

[5] In terms of the notation used by Altschuler, *op. cit.*, K is defined by $K^2 \equiv |\Gamma_\alpha|$.

or by

$$\begin{pmatrix}0 & 1\\ 1/k^2 & 0\end{pmatrix} * \begin{pmatrix}0 & Kk\\ Kk & 0\end{pmatrix} * \begin{pmatrix}0 & 1\\ e^{j2\Phi} & 0\end{pmatrix} * \begin{pmatrix}0 & e^{-j(\theta+\Phi)}\\ e^{-j(\theta+\Phi)} & 0\end{pmatrix}, \quad \text{for } k \geqq 1. \qquad (15\text{b})$$

The reciprocal elements ($S_{12}=S_{21}$) of (15) are recognized to be of the form of the elements described by (9) and (10). The remaining elements ($S_{12}=1$ or $S_{21}=1$) are "one-way" devices, in particular "one-way attenuators":

$$\begin{pmatrix}0 & k^2\\ 1 & 0\end{pmatrix} \text{ for } k \leqq 1 \quad \text{or} \quad \begin{pmatrix}0 & 1\\ 1/k^2 & 0\end{pmatrix} \text{ for } k \geqq 1, \qquad (16)$$

and "one-way phase shifters"

$$\begin{pmatrix}0 & e^{-j2\Phi}\\ 1 & 0\end{pmatrix} \quad \text{or} \quad \begin{pmatrix}0 & 1\\ e^{j2\Phi} & 0\end{pmatrix}. \qquad (17)$$

The associated network symbols, input-output relations, etc., are listed in Appendix II.

Based on the break-up given in (15) passive nonreciprocal two-ports can now be conveniently represented as shown in Fig. 4(a) or 4(b). Here $\theta_1=\beta l_1$, $\theta=\beta l$, and $\theta_2=\beta' l_2$. It is recognized that the particular break-up employed is only one of an infinite variety of possible ones; however, it is unique and meaningful in that each of the resulting elements performs a single well-defined function and is described by a single real number. Elements have been chosen in such a manner that (in the case of passive networks) all resulting elements are again necessarily passive. Clearly, when active networks are considered, elements such as "reciprocal amplifiers" or "one-way amplifiers" can be defined. As in the reciprocal case of the modified Wheeler network, all characteristic impedances are normalized to unity and the phase constant of the elements between transformers is arbitrarily assumed to be that associated with the in-port.

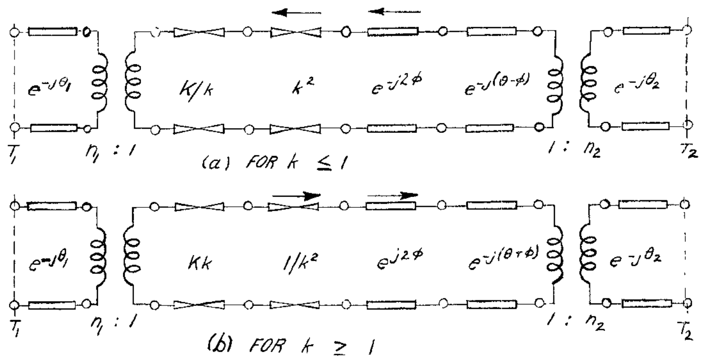
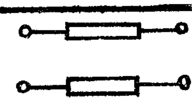
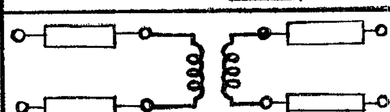
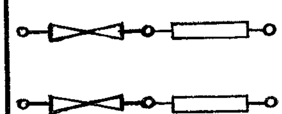
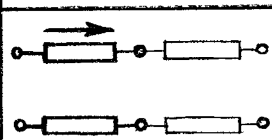
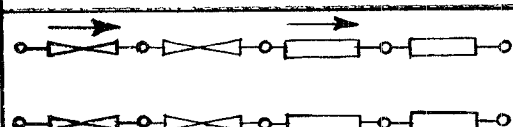


Fig. 4—Extension of the modified Wheeler network to the nonreciprocal case.

Each of the elements employed is particularly significant in context with certain practical microwave-structures in that close equivalences exist as shown in Table I (below). The elements of the network of Fig.

TABLE I

Idealized Element	Component Employed in Realization of Element	Representation of Component
Transmission line	Waveguide	
Ideal transformer	Lossless discontinuity	
Reflection coefficient transformer	Well matched practical attenuator	
One-way phase shifter	Well matched practical nonreciprocal phase shifter	
One-way attenuator	Well matched practical nonreciprocal attenuator	

4 consequently have physical meaning and lend themselves to a procedure for synthesizing physical two-ports with the properties of the network at the given frequency. The fact that the available microwave components are composite of our idealized elements is readily overcome in view of the commutivity of the bilaterally matched elements and the simple computations afforded by the star-product rule.

The break-up given here of the portion of the network between the transformers is to be looked upon as a systematic network description employing ideal elements which are defined in terms of their function and which may serve as a guide to synthesizing two-port structures with the properties of the network at the given frequency. With respect to such "synthesis," it is recognized that the reflection coefficient transformer, foı example, must be realized by means of the attenuation of a practical nonreciprocal attenuator in conjunction with the small reciprocal attenuation inherent in such components. Once the present point of view is adopted, it is of course adequate for many purposes to lump the three bilaterally matched elements (in Fig. 3) into a single network. The scattering matrix of this combination is

$$\begin{pmatrix} 0 & Kke^{-j(\Phi+\theta)} \\ K/ke^{-j(\Phi-\theta)} & 0 \end{pmatrix}. \tag{18}$$

The distinction between the "essential properties" of a two-port and those properties which may be altered by the connection of lossless reciprocal networks (tuners) at the in-port and out-port has already been mentioned in the introduction. These "essential properties" are explicitly exhibited by the reflection coefficient transformer, the one-way attenuator and the one-way phase shifter of the modified Wheeler network. In this connection both the forms in Fig. 3 and in Fig. 4 are of interest.

One sees immediately that the nonreciprocal behavior of a two-port is an "essential property." Moreover, when the form of Fig. 3 is employed the net nonreciprocal behavior of any number of arbitrary two-ports connected in tandem may be computed simply by multiplication of the amplifier phase shifters of the individual networks. The dissipation associated with a two-port is also an "essential property" in the following sense: the ultimate gain (minimum attenuation) of a two-port in either direction is that which results from the two dissipative elements in the modified Wheeler representation of Fig. 4. The minimum attenuation which may be achieved through any number of arbitrary two-ports connected in tandem, with tuners at every junction, is directly given by the product of these dissipative elements.

Measurement of the General Nonreciprocal Network

The measurement procedure indicated by Haus[1] holds here also. Briefly, in view of (4) and (7) the input reflection coefficient of the (reciprocal) modified Wheeler network, corresponding to a given termination, is identical to the input reflection coefficient of the general (nonreciprocal) two-port when it is similarly terminated. It follows that the measurement of the general two-port, by impedance techniques as already described elsewhere,[2] will yield l_1, n_1, K, l, n_2, and l_2, the parameters of the (reciprocal) modified Wheeler network.

Assuming that the scattering matrix of the general two-port in Fig. 1 is given by S, that of the nonreciprocal (ideal amplifier phase shifter) portion by S_n, and that of the reciprocal portion by S_r, it is readily shown (by the consideration of incident and reflected waves at the various terminal planes) that the respective mutual elements are related by

$$S_{r_{12}}S_{n_{12}} = S_{12}; \qquad S_{r_{21}}S_{n_{21}} = S_{21}. \tag{19}$$

It follows readily from the fact that $S_{r_{12}} = S_{r_{21}}$, from (19) and from the definition of the amplifier phase shifter, which requires $S_{n_{12}} = 1/S_{n_{21}}$, that

$$S_{n_{12}} = 1/S_{n_{21}} = \pm = \sqrt{S_{12}/S_{21}}. \tag{20}$$

In consequence standard transmission type measurements of S_{12}/S_{21} yield all the additionally necessary information. Defining

$$S_{12} = |S_{12}|\, e^{j\Phi_{12}}, \qquad S_{21} = |S_{21}|\, e^{j2\Phi_{21}} \tag{21}$$

one can readily identify k and Φ in (4) in terms of the measurable quantities as

$$k = \sqrt{|S_{12}|/|S_{21}|}; \qquad \Phi = (\Phi_{21} - \Phi_{12})/2 + n\pi, \quad n = 0, 1. \tag{22}$$

The fact that only the *ratio* of magnitudes and the *difference* between phase angles need be known to obtain k and Φ obviates the necessity for determining the exact values of either magnitudes or phase angles. These quantities evidently need be determined only to within an additive (for the phase angles) or a multiplicative (for the magnitudes) constant so that certain scale or equipment calibrations may be avoided. For measurement procedures see especially Macpherson[6] and also Pippin.[7]

Appendix I

Relationships Between Scattering Matrix and *ABCD* Matrix

The voltages, currents, incident and reflected waves, and matrix elements are defined in the standard manner shown in Fig. 5 where

[6] A. C. Macpherson, "Measurement of microwave nonreciprocal four-poles," Proc. IRE, vol. 43, p. 1017; August, 1955.

[7] J. E. Pippin, "Scattering matrix measurements on nonreciprocal microwave devices," Proc. IRE, vol. 44, p. 110; January, 1956.

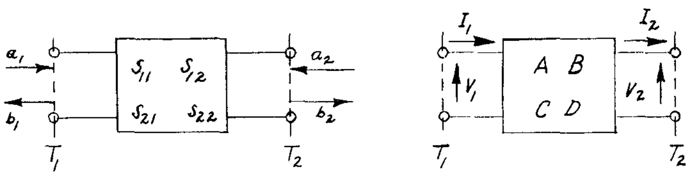


Fig. 5.

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \qquad \begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix}$$

For reciprocity: $S_{12} = S_{21}$ For reciprocity $AD - BC = 1$
The matrix elements are related as follows:

$$S_{11} = \frac{A+B-C-D}{A+B+C+D} \qquad A = \frac{S_{12}S_{21} + (1+S_{11})(1-S_{22})}{2S_{21}}$$

$$S_{22} = \frac{-A+B-C+D}{A+B+C+D} \qquad B = \frac{-S_{12}S_{21} + (1+S_{11})(1+S_{22})}{2S_{21}}$$

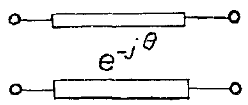
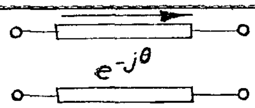
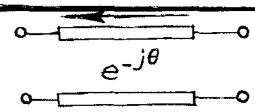
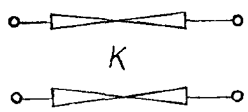
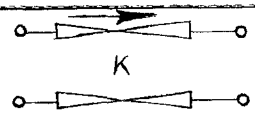
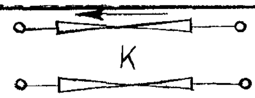
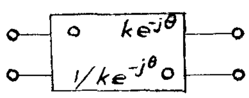
$$S_{12} = \frac{2(AD-BC)}{A+B+C+D} \qquad C = \frac{-S_{12}S_{21} + (1-S_{11})(1-S_{22})}{2S_{21}}$$

$$S_{21} = \frac{2}{A+B+C+D} \qquad D = \frac{S_{12}S_{21} + (1-S_{11})(1+S_{22})}{2S_{21}}.$$

Appendix II

TABLE II

Summary of Bilaterally Matched Elements

Name of Element	Scattering Matrix	Input-Output Relation	Network Symbol	Comments
Transmission Line	$\begin{pmatrix} 0 & e^{-j\theta} \\ e^{-j\theta} & 0 \end{pmatrix}$	$\Gamma_{in} = e^{-j2\theta}\,\Gamma_{out}$		Physical Length of Waveguide is ℓ; $\theta = \beta\ell$
One-Way Phase Shifter	$\begin{pmatrix} 0 & 1 \\ e^{-j\theta} & 0 \end{pmatrix}$	$\Gamma_{in} = e^{-j\theta}\,\Gamma_{out}$		Wave Propagating In Direction of Arrow Undergoes Phase Shift; Wave in Opposite Direction Remains Unaltered
One-Way Phase Shifter	$\begin{pmatrix} 0 & e^{-j\theta} \\ 1 & 0 \end{pmatrix}$	$\Gamma_{in} = e^{-j\theta}\,\Gamma_{out}$		
Reflectional Coefficient Transformer (Ideal Attenuator)	$\begin{pmatrix} 0 & K \\ K & 0 \end{pmatrix}$	$\Gamma_{in} = K^2\,\Gamma_{out}$		$0 \leq K \leq 1$
One-Way Attenuator	$\begin{pmatrix} 0 & 1 \\ K & 0 \end{pmatrix}$	$\Gamma_{in} = K\,\Gamma_{out}$		Wave Propagating In Direction of Arrow Undergoes Attenuation; Wave in Opposite Direction Remains Unaltered
One-Way Attenuator	$\begin{pmatrix} 0 & K \\ 1 & 0 \end{pmatrix}$	$\Gamma_{in} = K\,\Gamma_{out}$		
Ideal Amplifier Phase Shifter (Ratio Repeater)	$\begin{pmatrix} 0 & ke^{-j\Phi} \\ 1/ke^{-j\Phi} & 0 \end{pmatrix}$	$\Gamma_{in} = \Gamma_{out}$		Network is Active In One Direction And Passive in The Other